Math 10 Lecture Videos

Section 2.3:
Solving Linear Equations

PAUL ANDREW GORGONIO

OBJECTIVES:

1. Solve linear equations.
2. Solve linear equations containing fractions.
3. Solve linear equations containing decimals.
4. Identify equations with no solution or infinitely many solutions.
5. Solve applied problems using formulas.

Objective 1: Solve Linear Equations

Remember this? Linear Equation in One Variable

- Equation that can be written in the form **ax + b = c**
  - *a*, *b*, and *c* are <u>real numbers</u>
  - $a \neq 0$ (*a* is not equal to 0)

Given this, how do we solve this: $7x = 3(x + 8)$

1. Simplify the algebraic expression on each side.
2. Collect all the variable terms on one side and all the constant terms on the other side.
3. Isolate the variable and solve.
4. Check the proposed solution in the original equation.

Objective 1: Solve Linear Equations

Now, let's solve this: 7x = 3(x + 8)

$$7x = 3(x + 8)$$

$$7x = 3x + 24$$ Use the Distributive Property.

$$7x - 3x = 3x + 24 - 3x$$ Subtract $3x$ from both sides.

$$4x = 24$$ Simplify.

$$x = 6$$ Solve.

Check:

$$7(6) = 3(6 + 8)$$ Check the solution in the original equation.

$$42 = 3(14)$$

$$42 = 42$$ The true statement $42 = 42$ verifies 6 is the solution.

Objective 1: Solve Linear Equations

Example 2: -7x + 25 + 3x = 16 − 2x - 3

$$-7x + 25 + 3x = 16 - 2x - 3$$

$$-4x + 25 = 13 - 2x$$

Simplify the algebraic expression on each side.

$$-4x + 25 + 2x = 13 - 2x + 2x$$

Collect variable terms on one side and constant terms on the other side.

$$-2x + 25 = 13$$

Objective 1: Solve Linear Equations

Example 2: -7x + 25 + 3x = 16 − 2x - 3

$$-2x + 25 = 13$$

$$-2x + 25 - 25 = 13 - 25$$

$$-2x = -12$$

$$\frac{-2x}{-2} = \frac{-12}{-2}$$ Isolate the variable and solve.

$$x = 6$$

Objective 1: Solve Linear Equations

Example 3: 4(2x + 1) − 29 = 3(2x − 5)

$$4(2x + 1) - 29 = 3(2x - 5)$$

$$8x + 4 - 29 = 6x - 15$$

Simplify the algebraic expression on each side.

$$8x - 25 = 6x - 15$$

$$8x - 6x - 25 = 6x - 6x - 15$$

Collect variable terms on one side and constant terms on the other side.

$$2x - 25 = -15$$

Objective 1: Solve Linear Equations

Example 3: 4(2x + 1) − 29 = 3(2x − 5)

$$2x - 25 = -15$$

$$2x - 25 + 25 = -15 + 25$$

$$2x = 10$$

$$\frac{2x}{2} = \frac{10}{2}$$

Isolate the variable and solve.

$$x = 5$$

Objective 2: Solve linear equations containing fractions

Example 1: Solve and check $\dfrac{2x-1}{3} - \dfrac{x-2}{5} = \dfrac{x}{2}$

Step 1: Simplify the algebraic expressions on each side.

$$\frac{2x-1}{3} - \frac{x-2}{5} = \frac{x}{2}$$

$$30\left(\frac{2x-1}{3} - \frac{x-2}{5}\right) = 30\left(\frac{x}{2}\right)$$

Multiply both sides by Least Common Denominator: 30

$$\frac{30}{1}\left(\frac{2x-1}{3}\right) - \frac{30}{1}\left(\frac{x-2}{5}\right) = \frac{30}{1}\left(\frac{x}{2}\right)$$

Distributive Property

Objective 2: Solve linear equations containing fractions

Step 1: Simplify the algebraic expressions on each side.

$$\frac{30}{1}\left(\frac{2x-1}{3}\right) - \frac{30}{1}\left(\frac{x-2}{5}\right) = \frac{30}{1}\left(\frac{x}{2}\right)$$

$$\frac{10}{1}\left(\frac{2x-1}{1}\right) - \frac{6}{1}\left(\frac{x-2}{1}\right) = \frac{15}{1}\left(\frac{x}{1}\right) \qquad \text{Cancel}$$

$$10(2x-1) - 6(x-2) = 15x \qquad \text{Multiply}$$

$$20x - 10 - 6x + 12 = 15x \qquad \text{Distribute}$$

$$14x + 2 = 15x \qquad \text{Combine like terms}$$

Objective 2: Solve linear equations containing fractions

Step 2: Collect variable terms on one side and constant terms on other side.

$$14x - 14x + 2 = 15x - 14x$$

Subtract 14x from both sides

Step 3: Isolate the variable and solve

$$2 = x$$

Simplify

Objective 2: Solve linear equations containing fractions

Step 4: Check

$$\frac{2x-1}{3} - \frac{x-2}{5} = \frac{x}{2}$$ Original equation

$$\frac{2(2)-1}{3} - \frac{(2)-2}{5} \overset{?}{=} \frac{(2)}{2}$$ Replace *x* with 2

$$\frac{4-1}{3} - \frac{2-2}{5} \overset{?}{=} \frac{2}{2}$$ Simplify

$$\frac{3}{3} - \frac{0}{5} \overset{?}{=} \frac{2}{2}$$ Simplify

$$1-0 \overset{?}{=} 1$$ Simplify

$$1 = 1$$ Simplify

Objective 2: Solve linear equations containing fractions

Example 2: Solve $\dfrac{x}{4} = \dfrac{2x}{3} + \dfrac{5}{6}$

$$\frac{x}{4} = \frac{2x}{3} + \frac{5}{6}$$

$$12 \cdot \frac{x}{4} = 12\left(\frac{2x}{3} + \frac{5}{6}\right)$$

Begin by multiplying both sides of the equation by 12, the least common denominator

$$12 \cdot \frac{x}{4} = 12 \cdot \frac{2x}{3} + 12 \cdot \frac{5}{6}$$

$$3x = 8x + 10$$

Objective 2: Solve linear equations containing fractions

Example 2: Solve $\dfrac{x}{4} = \dfrac{2x}{3} + \dfrac{5}{6}$

$$3x = 8x + 10$$

$$3x - 8x = 8x - 8x + 10 \qquad \text{Subtract 8x in both sides}$$

$$-5x = 10$$

$$\frac{-5x}{-5} = \frac{10}{-5} \qquad \text{Isolate the variable x}$$

$$x = -2$$

Objective 3: Solve Linear Equations Using Decimals

Example 1: Solve 0.48x + 3 = 0.2 (x − 6)

$$0.48x + 3 = 0.2(x - 6)$$

$$0.48x + 3 = 0.2x - 1.2$$ Distributive Property

$$100(0.48x + 3) = 100(0.2x - 1.2)$$ Multiply both sides by 100 (or 10^2) to clear the decimals

Note: To clear an equation of decimals, multiply every term on both sides of the equation by a power of 10. The exponent on 10 will equal the greatest number of decimal places in the equation.

Objective 3: Solve Linear Equations Using Decimals

Example 1: Solve $0.48x + 3 = 0.2(x - 6)$

$$48x + 300 = 20x - 120$$

$$48x + 300 - 300 = 20x - 120 - 300$$

$$48x = 20x - 420$$

$$48x - 20x = 20x - 20x - 420$$

$$28x = -420$$

$$\frac{28x}{28} = -\frac{480}{28}$$

$$x = -15$$

Objective 4: Identify solutions with no solution or infinitely many solutions.

Example 1: Solve 2x = 2(x + 3)

$$2x = 2(x + 3)$$

$$2x = 2x + 6$$ Distributive Property

$$2x - 2x = 2x - 2x + 6$$ Subtract 2x from both sides

Inconsistent No Solution $$0 = 6$$ Simplify. False, $0 \neq 6$

Note: Some equations are not true for any real number. Such equations are call **inconsistent** equations or **contradictions**.

Objective 4: Identify solutions with no solution or infinitely many solutions.

Types of Equations	Definitions
Identity	An equation that is true for *all* real numbers
Conditional	An equation that is *not* an identity but is true for *at least one* real number
Inconsistent (contradiction)	An equation that is *not true* for *any* real number

Objective 4: Identify solutions with no solution or infinitely many solutions.

Example 2: Solve 5 + 4x = 9x + 5

$$5 + 4x = 9x + 5$$

$$5 - 5 + 4x = 9x + 5 - 5 \qquad \text{Subtract 5 from both sides}$$

$$4x = 9x \qquad \text{Simplify}$$

$$4x - 4x = 9x - 4x \qquad \text{Subtract } 4x \text{ from both sides}$$

$$0 = 5x$$

Objective 4: Identify solutions with no solution or infinitely many solutions.

Example 2: Solve 5 + 4x = 9x + 5

$$0 = 5x \qquad \text{Simplify}$$

$$\frac{0}{5} = \frac{5x}{5} \qquad \text{Divide both sides by 5}$$

$$0 = x \qquad \text{Simplify}$$

The original equation is only true when x = 0.
Therefore, it is a Conditional Equation.

Objective 4: Identify solutions with no solution or infinitely many solutions.

Example 3: Solve 3 + 2x = 3(x + 1) - x

$$3 + 2x = 3(x + 1) - x$$

$$3 + 2x = 3x + 3 - x \qquad \text{Distribute the 3.}$$

$$\underline{3 + 2x} = \underline{2x + 3} \qquad \text{Simplify.}$$

Note: The left hand side (LHS) is equal to the right hand side (RHS) of the equation, this is an **IDENTITY** and is always true for all x.

Solve Applied Problems using Formulae

The formula $p = 15 + \dfrac{5d}{11}$ describes the pressure of sea water p, in pounds per square foot, at a depth d feet below the surface. At what depth is the pressure 30 pounds per square foot?

Substitute the given pressure into the equation for p, $p = 30$. The equation becomes:

$$30 = 15 + \dfrac{5d}{11}$$

Objective 5:
Solve Applied Problems using Formulae

$$30 = 15 + \frac{5d}{11}$$

$$30(11) = \left(15 + \frac{5d}{11}\right)11$$ Multiply both sides by 11 the LCD.

$$330 = 165 + 5d$$ Simplify.

$$330 - 165 = 165 - 165 + 5d$$ Subtract 165 from both sides.

Objective 5:
Solve Applied Problems using Formulae

$165 = 5d$ Simplify.

$$\frac{165}{5} = \frac{5d}{5}$$ Divide both sides by 5.

$d = 33$ Simplify.

The depth where the pressure is 30 pounds per square foot is 33 feet.

OBJECTIVES:

1. Solve linear equations. ✓
2. Solve linear equations containing fractions. ✓
3. Solve linear equations containing decimals. ✓
4. Identify equations with no solution or infinitely many solutions. ✓
5. Solve applied problems using formulas. ✓